Chapter 3

Dislocation

3.1 Shear stress and slip

If tensile stress is continuously applied to the metal specimen, the average distance between the metal atoms continues to increase, and at some point, the metal undergoes plastic deformation. The stress level for this point is the yield strength. In the case of a typical metal, the yield stress is about several hundreds MPa. However, when a stress of less than 1 MPa, which is much lower than that, is sometimes applied to the metal specimen, as shown in Fig. 3.1, such a layered pattern is formed and the metal is deformed. This deformation is called slip and the line in Fig. 3.1 is called the slip line. Slip refers to the sliding of a material along a specific surface. Although this phenomenon has been observed for a long time, explanations are relatively recent. In Fig. 3.3(b), the separation of two planes are approximately two atomic radii therefore, the shear strain is

$$\gamma \simeq \frac{a}{2a} = \frac{1}{2}$$

Without consideration of plasticity, we have

$$\mu = \frac{\tau}{\gamma}$$

where μ is shear modulus and τ is the shear stress. The shear modulus of Mg is 17.2 GPa, the shear stress at saddle point is

$$\tau = \frac{17.2}{2} = 8.6\,\mathrm{GPa}$$

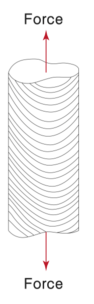

Figure 3.1: Slip lines on magnesium crystal.

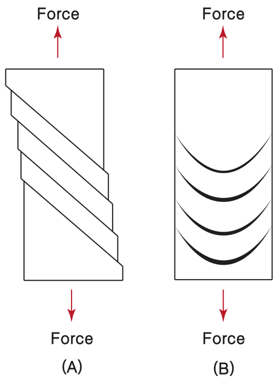

Figure 3.2: Schematic view of slip lines. (a) side view (b) front view.

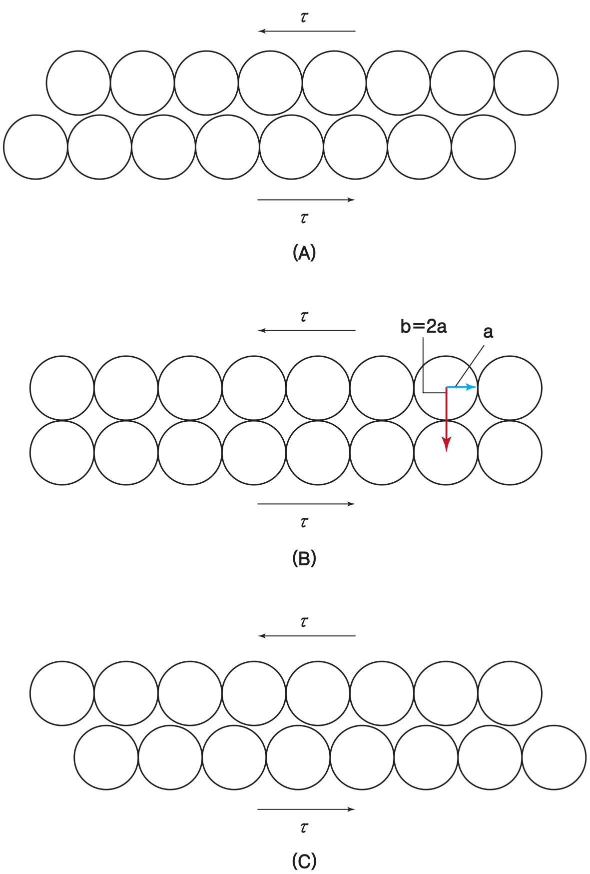

Figure 3.3: (a) Initial position of the atoms on silp plane. (b) At saddle point. (c) Final position after shear by one atomic distance.

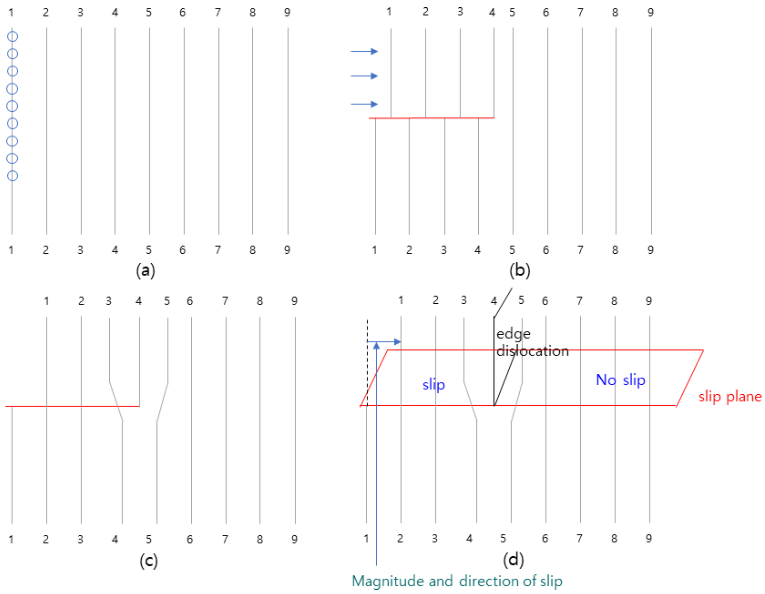

Figure 3.4: Schematic representation of edge dislocation generation.

Experimentally, the shear stress to be applied for shear is 0.7 MPa, which differs by four order of magnitudes. We can imagine that something is going on there. It doesn't seem easy to just pass the saddle point, so I thought a lot about other mechanisms.

3.2 Kinds of Dislocation

3.2.1 Edge dislocation and Screw Dislocation

The basic assumption to explain that phenomenon is that materials are not perfect. There is a defect in it. Not all atomic planes are perfect, some atomic planes are either partial or truncated. We will call this case dislocation from now on. With the development of electron microscopy technology, this dislocation can also be observed experimentally. This existence has been experimentally proven. In Fig.3.4(a), there are nine atomic planes with no line defects in it. The cut plane (red line) is inserted and all bonds are broken across the cut plane in Fig.3.4(b). When stress is applied to direction by blue arrow, then four half atomic planes migrate to the right direction. When half planes migrate by one atomic distance, the atomic plane is connected again, however two half planes (plane 1 below half and plane 4 above half) are remained as half planes. The slip plane is the extended plane of cut plane and the slip region means that the atomic layers are partial or connected plane with two different original numbering, such as 1-2 and 2-3 and so on. In no slip region, all atomic planes have been intact. The edge dislocation is the boundary between slip region and no slip region. The representation of edge dislocation in 3D is shown in Fig. 3.5. The dislocation line is defined along edge of the extra half plane and it is represented by $\perp$ symbol. The magnitude of the plane slip is represented by Burgers vector, **b**. Once we have the atomic circuit including edge dislocation in Fig. 3.6, With the Start atom (S), then go down to 1D, 2D, 3D and 4D and then move to right side and we have 4D1R to 4D3R. Move to the upside four times, then

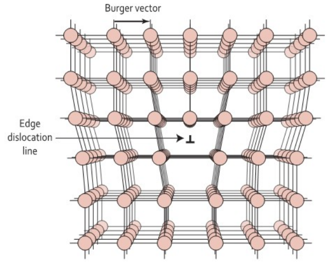

Figure 3.5: Atomic arrangement including edge dislocation.

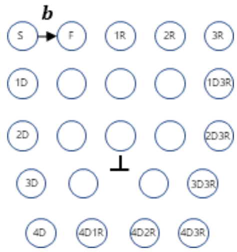

Figure 3.6: Atomic arrangement including edge dislocation.

we have 3R and move two left side three times and the atom is final atom (F). When there is no dislocation, S atom have to be same as F atom. If it is not same, then the vector from S atom to F atom is the Burgers vector, **b**. For the case of edge dislocation, the Burgers vector is perpendicular to the dislocation line. Please be cautious that one have to rotate along counter-clockwise direction. When you rotate along clockwise direction, Burgers vector is vector from F atom to S atom. We found that for a edge dislocation,

$$\mathbf{b} \perp \mathbf{t}$$

where **t** is the direction vector for dislocation line. There is another form of dislocation, which is a screw dislocation. In Fig. 3.7, even without a half plane, the dislocation line and the Burgers vector are parallel, and it can be understood as rotating at a certain angle with respect to the axis of rotation perpendicular to the slip plane.

$$\mathbf{b} \parallel \mathbf{t}$$

3.2.2 Mixed dislocation

Typically, in real materials, we frequently encounter mixed dislocation, at point A in Fig. 3.9 the dislocation is a screw dislocation and it becomes edge dislocation at point B.

3.3 Dislocation(Vacancy) loop and Prismatic loop

There are cases where the slip region creates a closed space. There are two major cases: a dislocation loop and a prismatic loop. Assume the Burgers vector indicates

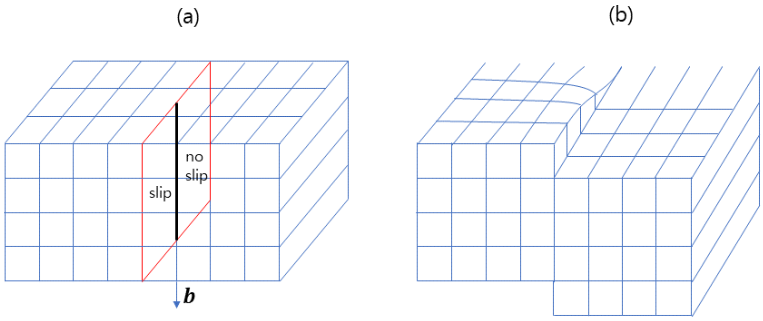

Figure 3.7: Schematic illustration of screw dislocation

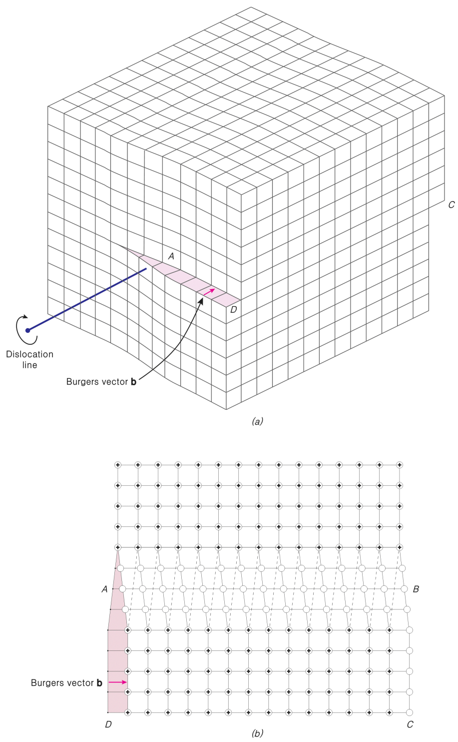

Figure 3.8: Better representation of screw dislocation.

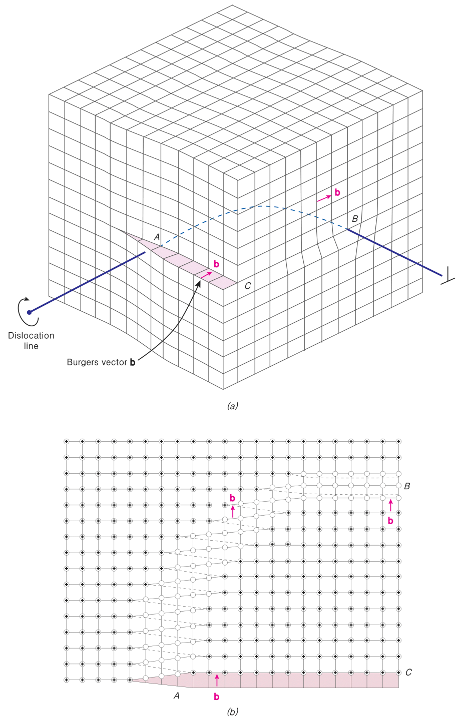

Figure 3.9: Representation of mixed dislocation.

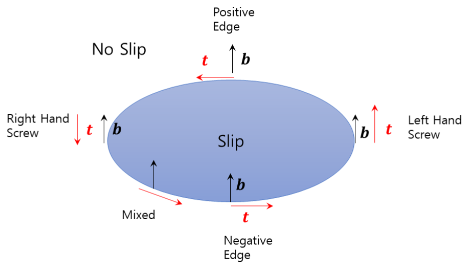

Figure 3.10: Schematic illustration of dislocation loop

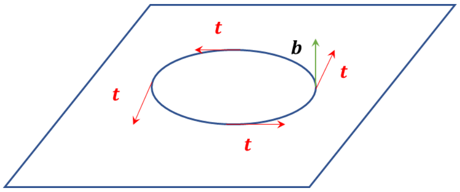

Figure 3.11: Schematic illustration of prismatic dislocation loop

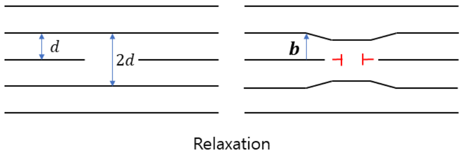

Figure 3.12: Schematic illustration of vacancy loop

upside, dislocation line is the tangential line of the loop, therefore, it is combination of edge and screw dislocations. When turned clockwise, when **t** precedes **b**, it is called a positive edge, and when it follows, it is called a negative edge. If **t** and **b** are parallel, the left hand screw, when the angle is 180°, it is called a right hand screw. A dislocation loop consists of negative edge, left hand screw, positive edge, right hand screw. The another type of loop is a prismatic dislocation loop in Fig. 3.11, and it is a structure mainly found in metals such as Fe, which are mainly irradiated with neutrons. As will be discussed in detail later, when neutrons are irradiated, many defects out of their original site called self interstitial atoms and vacancies are created, which sometimes form a partial plane or a vacancy cluster between atomic planes in the form of a two-dimensional disk. At this time, Burgers vector is perpendicular to the plane, and in this case, Burgers vector and dislocation line are perpendicular to all regions of the loop. Therefore, the entire loop has the edge dislocation. A vacancy loop in Fig. 3.12 is an example of prismatic loop and we can easily find the location of edge dislocation in it.

3.4 Partial dislocation

The movement of dislocations along the slip plane is in fact achieved by movement of atoms in the opposite direction. When looking at the movement of dislocation along the (111) plane of FCC in Fig. 3.13, the direction of movement is indicated by Burgers vector, however, sometimes, it is not easy because it requires large amount of lattice distortion in this process. If a single unit dislocation break down into a pair of partial dislocations, the vectors **c** and **d** are shown in Fig. 3.14. The Burgers

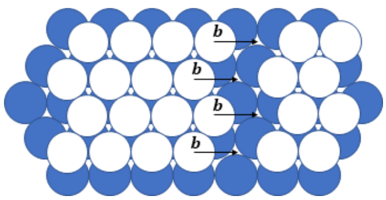

Figure 3.13: A total dislocation in a FCC lattice.

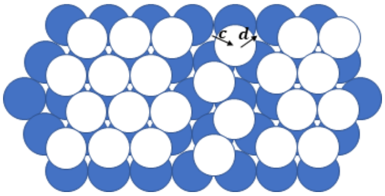

Figure 3.14: Partial dislocation in a FCC lattice.

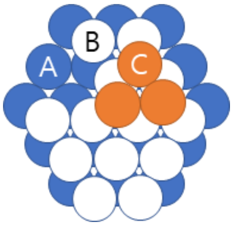

Figure 3.15: Stacking of atomic layers A, B and C

vector on (111) plane of FCC is

$$\frac{1}{2}[\bar{1}10]$$

and two partial vectors are

$$\frac{1}{6}[\bar{1}2\bar{1}] \qquad \frac{1}{6}[\bar{2}11]$$

We confirm that

$$\frac{1}{2}[\bar{1}10] = \frac{1}{6}[\bar{1}2\bar{1}] + \frac{1}{6}[\bar{2}11]$$

The partial dislocation discussed above is called as Shockley partial. In the case of FCC, in Fig. 3.15, it is a sequence of ABABAB ... layers where A is followed by B and then A again. An atom that comes to B or C in the order in which A comes is called a stacking fault. As shown in Fig 3.14, to generate Shockley partial, generation of a stacking fault is inevitable. Therefore, the larger the stacking fault energy (SFE), the less likely it is to generate a Shockley partial, but a perfect dislocation. In the case of Shockley partial, cross slip does not occur because the direction of dislocation is changed. This has a significant impact on the mechanical properties of the material, as we will see later. As shown in Fig. 3.16, when half plane present above part, since the atomic density is high, compressive strain field presents. On the other hand, atomic arrangement is relatively loose, therefore, tensile strain field presents.

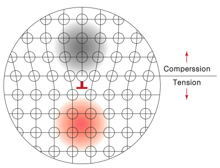

Figure 3.16: Regions of compression (dark) and tension (colored) located around an edge dislocation.

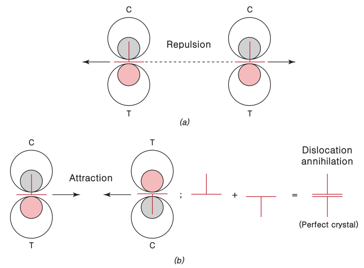

Figure 3.17: (a) Two edge dislocations of the same sign and lying on the same slip plane exert a repulsive force on each other; C and T denote compression and tensile regions, respectively. (b) Edge dislocations of opposite sign and lying on the same slip plane exert an attractive force on each other. Upon meeting, they annihilate each other and leave a region of perfect crystal.

3.5 Stress field by a dislocation

Let's see an qualitative explanation in Fig. 3.17. Edge dislocations with the same sign repel each other. Repulsive force is applied because compressive strain zones do not want to overlap each other and tensile strain zones do not want to be close to each other. On the other hand, attractive force is applied to the compressive strain zone and the tensile strain zone, and even when two edge dislocations with different signs meet, the dislocation disappears while forming a perfect plane. This is called dislocation annihilation. In order to quantitatively explain this interaction, it is necessary to find the force acting by dislocation. Let's take a look at this part.

3.5.1 Basic elasticity theory

The displacement of a point in a strained body from its position in the unstrained state is represented by

$$\mathbf{u} = [u_x, u_y, u_z]$$

Then the strain is defined by

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial u_y}{\partial y} \qquad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)$$

$$\varepsilon_{zx} = \varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

With Lamé constant λ and μ,

$$\sigma_{xx} = 2\mu\varepsilon_{xx} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$$

$$\sigma_{yy} = 2\mu\varepsilon_{yy} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$$

$$\sigma_{zz} = 2\mu\varepsilon_{zz} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$$

$$\sigma_{xy} = 2\mu\varepsilon_{xy} \qquad \sigma_{yz} = 2\mu\varepsilon_{yz} \qquad \sigma_{zx} = 2\mu\varepsilon_{zx}$$

3.5.2 Mechanical Equilibrium

For deriving the differential equations of equilibrium one has to apply Newton's second law to a small rectangular volume element, $\delta x \delta y \delta z$, under externally imposed forces $\mathbf{F}$. Neglecting presence of body force for simplicity. Then the Newton's second law yield

$$
\begin{aligned}
m\frac{\partial^2 u_x}{\partial t^2} &= \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\delta x - \sigma_{xx}\right)\delta y \delta z + \left(\sigma_{xy} + \frac{\partial \sigma_{xy}}{\partial y}\delta y - \sigma_{xy}\right)\delta x \delta z \\
&\quad + \left(\sigma_{xz} + \frac{\partial \sigma_{xz}}{\partial z}\delta z - \sigma_{xz}\right)\delta x \delta y \\
m\frac{\partial^2 u_y}{\partial t^2} &= \left(\sigma_{yy} + \frac{\partial \sigma_{y}}{\partial y}\delta y - \sigma_{yy}\right)\delta x \delta z + \left(\sigma_{yx} + \frac{\partial \sigma_{yx}}{\partial x}\delta x - \sigma_{yx}\right)\delta y \delta z \\
&\quad + \left(\sigma_{yz} + \frac{\partial \sigma_{yz}}{\partial z}\delta z - \sigma_{yz}\right)\delta y \delta x \\
m\frac{\partial^2 u_z}{\partial t^2} &= \left(\sigma_{zz} + \frac{\partial \sigma_{zz}}{\partial z}\delta z - \sigma_{zz}\right)\delta x \delta y + \left(\sigma_{zx} + \frac{\partial \sigma_{zx}}{\partial x}\delta x - \sigma_{zx}\right)\delta z \delta y \\
&\quad + \left(\sigma_{zy} + \frac{\partial \sigma_{zy}}{\partial y}\delta y - \sigma_{zy}\right)\delta z \delta x
\end{aligned}
$$

Under the equilibrium,

$$
\begin{aligned}
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= 0 \\
\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} &= 0 \\
\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} &= 0
\end{aligned}
$$

In short, we have

$$
\frac{\partial \sigma_{ij}}{\partial j} = 0 \tag{3.1}
$$

which is called the equation of mechanical equilibrium.

3.5.3 Stress field in cylindrical coordinate

When describing the stress field of a dislocation, it is sometimes more convenient to describe it in cylindrical coordinates.

$$
\begin{aligned}
\sigma_{rr} &= \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\sigma_{xy}\sin\theta\cos\theta \\
\sigma_{\theta\theta} &= \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\sigma_{xy}\sin\theta\cos\theta \\
\sigma_{r\theta} &= (\sigma_{yy} - \sigma_{xx})\sin\theta\cdot\cos\theta + \sigma_{xy}(\cos^2\theta - \sin^2\theta) \\
\sigma_{zz} &= \sigma_{zz} \\
\sigma_{rz} &= \sigma_{xz}\cos\theta + \sigma_{yz}\sin\theta \\
\sigma_{\theta z} &= -\sigma_{xz}\sin\theta + \sigma_{yz}\cos\theta
\end{aligned}
$$

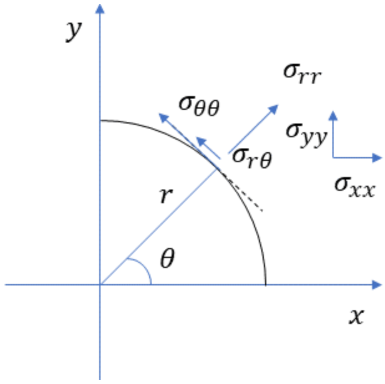

Figure 3.18: Stress field in cylindrical coordinate

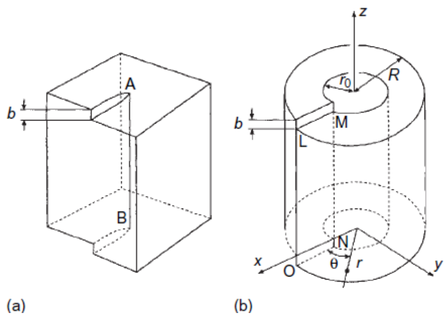

Figure 3.19: (a) Screw dislocation AB formed in a crystal. (b) Elastic distortion of a cylindrical tube simulating the distortion produced by the screw dislocation in (a).

3.5.4 Stress field of screw dislocation

We assume that there is no displacements in the x and y directions,

$$u_x = u_y = 0$$

$$u_z = \frac{b\theta}{2\pi} = \frac{b}{2\pi}\arctan(y/x)$$

then we have

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{yx} = 0$$

Since

$$\frac{\partial u_z}{\partial x} = \frac{b}{2\pi} \times \frac{1}{1+(y/x)^2} \times \frac{-y}{x^2} = -\frac{b}{2\pi}\frac{y}{x^2+y^2} = -\frac{b}{2\pi}\frac{\sin\theta}{r}$$

$$\frac{\partial u_z}{\partial y} = \frac{b}{2\pi} \times \frac{1}{1+(y/x)^2} \times \frac{1}{x} = \frac{b}{2\pi}\frac{x}{x^2+y^2} = \frac{b}{2\pi}\frac{\cos\theta}{r}$$

Therefore,

$$\varepsilon_{zx} = \varepsilon_{xz} = -\frac{b}{4\pi}\frac{y}{x^2+y^2} = -\frac{b}{4\pi}\frac{\sin\theta}{r}$$

$$\varepsilon_{zy} = \varepsilon_{yz} = \frac{b}{4\pi}\frac{x}{x^2+y^2} = \frac{b}{4\pi}\frac{\cos\theta}{r}$$

With Hooke's law,

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yx} = 0$$

$$\sigma_{zx} = \sigma_{xz} = -\frac{\mu b}{2\pi}\frac{y}{x^2+y^2} = -\frac{\mu b}{2\pi}\frac{\sin\theta}{r}$$

$$\sigma_{zy} = \sigma_{yz} = \frac{\mu b}{2\pi}\frac{x}{x^2+y^2} = \frac{\mu b}{2\pi}\frac{\cos\theta}{r}$$

In the cylindrical coordinates, we have

$$\sigma_{rz} = \sigma_{xz}\cos\theta + \sigma_{yz}\sin\theta = 0$$

$$\varepsilon_{rz} = \varepsilon_{xz}\cos\theta + \varepsilon_{yz}\sin\theta = 0$$

$$\sigma_{\theta z} = -\sigma_{xz}\sin\theta + \sigma_{yz}\cos\theta = \frac{\mu b}{2\pi r}$$

$$\varepsilon_{\theta z} = -\varepsilon_{xz}\sin\theta + \varepsilon_{yz}\cos\theta = \frac{b}{4\pi r}$$

3.5.5 Stress field of edge dislocation

For edge dislocations, unlike screw dislocations, it is difficult to express displacement in a simple way, so we will describe the stress field directly.

$$\sigma_{xx} = -Dy\frac{3x^2+y^2}{(x^2+y^2)^2} \qquad \sigma_{yy} = Dy\frac{x^2-y^2}{(x^2+y^2)^2} \qquad \sigma_{zz} = \nu(\sigma_{xx}+\sigma_{yy})$$

$$\sigma_{xy} = \sigma_{yx} = Dx\frac{x^2-y^2}{(x^2+y^2)^2} \qquad \sigma_{xz} = \sigma_{zx} = \sigma_{yz} = \sigma_{zy} = 0$$

where

$$D = \frac{\mu b}{2\pi(1-\nu)}$$

The general trend is

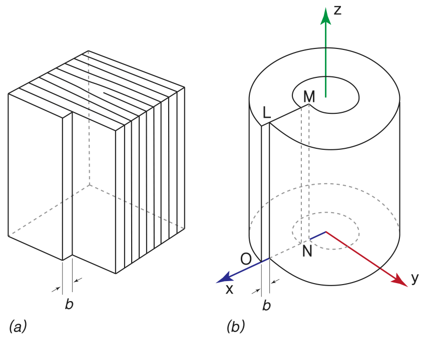

Figure 3.20: (a) Edge dislocation formed in a crystal. (b) Elastic distortion of a cylindrical ring simulating the distortion produced by the edge dislocation in (a).

1. Above the edge ($x = 0, y > 0$), pure compression
2. Below the edge ($x = 0, y < 0$), pure tension
3. Along the slip plane ($y = 0$), pure shear

When $\mathbf{b} = [100]$, $\mathbf{t} = [001]$, we can represent the stress state as shown in Fig. 3.21.

3.5.6 Strain energy of a dislocation

Dislocation causes deformation of the lattice, which in turn generates extra energy, which is called strain energy. The strain energy E_{total} is divided into two parts, core energy(E_{core}) and elastic energy (E_{el}).

$$E_{\text{total}} = E_{\text{core}} + E_{\text{el}}$$

The core energy is not easy to calculate on a continuum scale and is usually calculated using quantum mechanics, etc. The evaluation of the core energy is beyond scope of the lecture. The strain energy by a screw dislocation is can be obtained by work done in displacing the faces of the cut LMNO by b in Figs. 3.19 and 3.20. Since the strain energy is

$$dE_{\text{el}} = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}dV$$

in an elemental volume dV. In the cylindrical coordinate,

$$dV = 2\pi r dr dh$$

for unit axis length $dh = 1$, we have

$$dE_{\text{el}}(\text{screw}) = 2\pi r \times \frac{1}{2}dr\left(\sigma_{\theta z}\varepsilon_{\theta z} + \sigma_{z\theta}\varepsilon_{z\theta}\right) = 4\pi r dr \mu\left(\varepsilon_{\theta z}\right)^2 = \frac{\mu b^2}{4\pi r}dr$$

proceed to

$$E_{\text{el}}(\text{screw}) = \frac{\mu b^2}{4\pi}\int_{r_0}^{R}\frac{dr}{r} = \frac{\mu b^2}{4\pi}\ln\left(\frac{R}{r_0}\right)$$

It is bit difficult to derive, but the result is rather simple and we have

$$E_{\text{el}}(\text{edge}) = \frac{\mu b^2}{4\pi(1-\nu)}\ln\left(\frac{R}{r_0}\right)$$

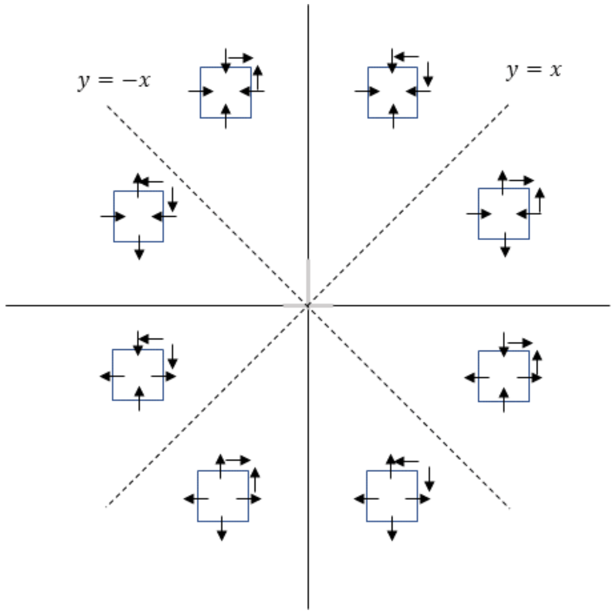

Figure 3.21: Stress state in the vicinity of a edge dislocation when $\mathbf{b} = [100]$, $\mathbf{t} = [001]$

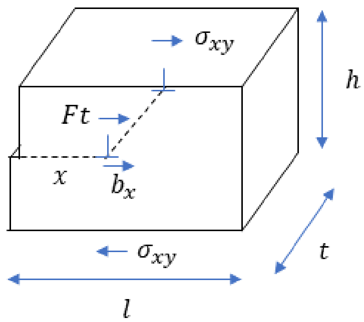

Figure 3.22: An edge dislocation displaced x with stress σ_{12}

Since

$$1 - \nu < 1$$

therefore, we can expect that elastic energy of edge dislocation is larger than that of screw dislocation.

3.5.7 Force on dislocation: Peach-Koehler equation

The average displacement along x direction, u_x in Fig. 3.22 is

$$u_x = \frac{xt}{lt} b_x$$

The external force generated by shear stress σ_{xy} is $\sigma_{xy} lt$, the work for slip is

$$W = \sigma_{xy} lt u_x = \sigma_{xy} x t b_x = F_x t x$$

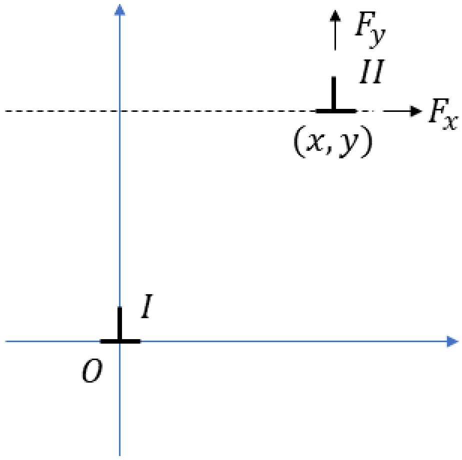

Figure 3.23: Dislocation array when $\mathbf{b} = [b00]$ and $\mathbf{t} = [001]$

where F_x is the force along x direction on the dislocation on unit length of dislocation. Therefore, we have

$$F_x = \sigma_{xy} b_x$$

Given a dislocation line $\mathbf{t}$ and a Burgers vector $\mathbf{b}$ in general, finding the force can be generalized with more geometrical considerations, and this is called the Peach-Koehler equation. The force per unit length on a dislocation in the presence of applied stress σ_{ij}, we have

$$\mathbf{F} = (\sigma_{ij} \cdot \mathbf{b}) \times \mathbf{t}$$

where $\mathbf{t}$ is an unit vector. To describe in more detail

$$\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ G_x & G_y & G_z \\ t_x & t_y & t_z \end{vmatrix}$$

where

$$G_x = \sigma_{xx} b_x + \sigma_{xy} b_y + \sigma_{xz} b_z$$
$$G_y = \sigma_{yx} b_x + \sigma_{yy} b_y + \sigma_{yz} b_z$$
$$G_z = \sigma_{zx} b_x + \sigma_{zy} b_y + \sigma_{zz} b_z$$

For the case in Fig. 3.23, force per unit length applied on dislocation II by dislocation I is

$$G_x = \sigma_{xx} b \qquad G_y = \sigma_{yx} b \qquad G_z = \sigma_{zx} b$$

then

$$\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sigma_{xx} b & \sigma_{yx} b & \sigma_{zx} b \\ 0 & 0 & 1 \end{vmatrix}$$

therefore

$$F_x = \sigma_{yx} b \qquad F_y = -\sigma_{xx} b \qquad F_z = 0$$

F_x is the force in the glide direction and F_y is the force perpendicular to the glide plane. When the case of dislocation II is negative edge, the sign of force have to be reversed. Therefore, we have

$$F_x = \frac{\mu b b'}{2\pi(1-\nu)} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2} = \frac{\mu b b'}{2\pi(1-\nu) r} \cos\theta \cos 2\theta \tag{3.2}$$

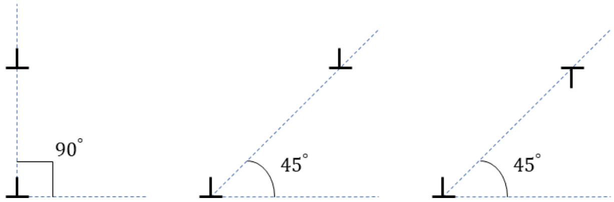

Figure 3.24: Three stable configurations of two edge dislocations.

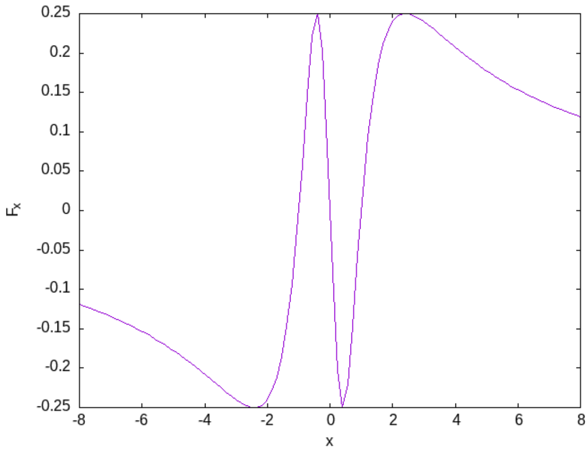

Figure 3.25: Plot of F_x with respect to $Gbb'y/2\pi(1-\nu)$. Assume that $y = 1$.

$$F_y = \frac{\mu bb'}{2\pi(1-\nu)} \frac{y(3x^2+y^2)}{(x^2+y^2)^2} = \frac{\mu bb'}{2\pi(1-\nu)r} \sin\theta\big(2+\cos 2\theta\big)$$

We can find the stable position for two edge dislocations force becomes 0 when

$$x = 0 \qquad y \to \infty$$

Three stable configurations of two edge dislocations are shown in Fig. 3.24. The force per unit length F_x, we have the plot in Fig. 3.25. In Fig. 3.26,

$$F_x = F_r \cos\theta - F_\theta \sin\theta$$

$$F_y = F_r \sin\theta - F_\theta \cos\theta$$

Therefore,

$$F_r = F_x \cos\theta + F_y \sin\theta = \frac{\mu bb'}{2\pi(1-\nu)r}$$

$$F_\theta = F_y \cos\theta - F_x \sin\theta = \frac{\mu bb' \sin 2\theta}{2\pi(1-\nu)r}$$

From Eq. 3.2, when $bb' > 0$, when $0 < \theta < \pi/4$, $F_x > 0$, it means the dislocation tends to move away. When $\pi/4 < \theta < \pi/2$, $F_x < 0$, it means the dislocation tends to converge. Therefore, the position $\theta = \pi/4$ is the unstable point, because, even a slight fluctuation causes it to deviate from its original position. On the other hand, when $\pi/2 < \theta < 5\pi/4$, $F_x > 0$, it means that the dislocation at $\pi/4 < \theta < 5\pi/4$ tends to converge to $\theta = \pi/2$, which means the point at $\theta = \pi/2$ is the stable position. The moving direction of the edge dislocation by the force exerted by parallel edge dislocation at origin is visualized in Fig. 3.27.

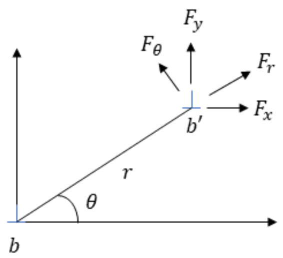

Figure 3.26: Force between two parallel edge dislocations.

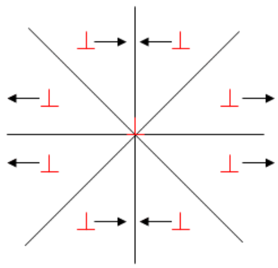

Figure 3.27: The moving direction of the edge dislocation by the force exerted by parallel edge dislocation at origin.

3.6 Interaction between Dislocations

A straight dislocation line can have a break in it in Fig.3.28:

1. A jog moves it out of the current slip plane. (→ to a parallel one)
2. A kink leaves the dislocation on the slip plane.

The Jog and the Kink can be considered as a defect in a dislocation line. Jogs and Kinks can be produced by intersection of straight dislocations. The presence of a jog in a dislocation line increases the energy of the crystal. The energy of a jog per unit length is less than that for the dislocation (as this lies in the distorted region near the core of the dislocation). This energy is about $0.5 - 1.0\,\mathrm{eV}(\sim 10^{-19}\mathrm{J})$ for metals. The energy of the jog can be derived by

$$E_{\text{jog}} = \alpha G b_1^2 b_2$$

where b_1 is the Burgers vector of the dislocation and b_2 is the length of the jog and α is the constant somewhere between 0.5 to 1. When b_2 is comparable to the $0.5b_1$ the energy is frequently assumed by

$$E_{\text{jog}} = 0.2Gb^3 \tag{3.3}$$

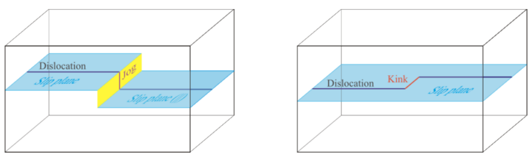

Figure 3.28: Schematic illustrations of jog and kink.

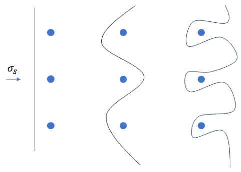

Figure 3.29: Schematics of a dislocation moving through an array of obstacles in its glide plane.

with omitting subscript. Since the energy of ordinary dislocation per unit length is Gb^2, the factor 0.2 in Eq. 3.3 arises from the much-reduced range of the stress field due to the jog compared to that around an ordinary dislocation. When there are miltiple jogs are in the system and their spacing is L, the work performed by the stress is

$$W = Lb^2 \left(\sigma_s^{\text{crit}}\right)_{\text{jog}}$$

Then the critical stress to move the screw dislocation by forcing the jogs to climb is

$$\left(\sigma_s^{\text{crit}}\right)_{\text{jog}} \simeq 0.2\frac{Gb}{L}$$

By similar logic, when a dislocation is impeded in its movement by an obstacle, the amount of shear stress that must be applied to overcome the obstruction is given by

$$\left(\sigma_s^{\text{crit}}\right)_{\text{jog}} \simeq 2\frac{Gb}{L} \tag{3.4}$$